# Self-focusing and merging of two copropagating laser beams in underdense plasma

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The propagation of two laser beams copropagating in underdense plasma has been studied numerically by solving their coupled envelope equations. It shows that two beams can merge each other, or split into three beams, or propagate with unstable trajectories, depending upon their power and initial beam separation. During the merging process, strong emission of radiation is observed. It also shows that the density cavitation channels due to the transverse ponderomotive force of the beams tend to trap them inside and prevent them from merging each other.

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Optical spatial solitons have been attracting continued interest since 1960's [1]. A variety of nonlinear optical materials, including Kerr media, photorefractive materials, quadratic nonlinear material, saturable nonlinear media, and plasmas, etc., can support the self-focusing/self-trapping of light beams. In some particular case, it appears as optical spatial solitons, provided that the diffraction of light beams is exactly balanced by the nonlinear focusing effect. Recently, there has been much interest in the interaction between such kind of spatial solitons as well as two or more light beams, which are launched into these nonlinear media in directions either parallel to each other or at some crossing angles. Compared with the interaction of one-dimensional temporal solitons in optical fibers, the interaction between two optical spatial solitons exhibits distinctive features such as mutual attraction/beam fusion [2–4], repulsion, beam fission [5,6], and beam spiraling [7,8], etc. This mutual interaction occurs intensively around the critical power or above for self-focusing/self-trapping, which ranges from a few microwatt ($\mu$W) only in photorefractive materials biased with some external dc field [9] up to terawatt (TW) or above in plasmas [10,11].

Energy transfer between interacting laser beams in plasma directly addresses fundamental aspect of laser plasma interaction and is also relevant to laser-driven inertial confined fusion [12,13]. In plasma, usually the nonlinear coupling between the interacting beams comes from the relativistic mass correction and the plasma density modification owing to the ponderomotive push on electrons [11], if one neglects beam coupling through stimulated Brillouin and Raman scattering and ion-acoustic wave [12,13]. For examples, in the geometry of counterpropagating laser beams, Shvets and Pukhov has proposed the electromagnetically induced guiding owing to the formation of a high-amplitude density grating produced by the interference of the two beams [14]; for two spatially separated intense laser beams copropagating in underdense plasma, Ren *et al.* have observed the mutual beam attraction and beam spiraling in recent three-dimensional particle-in-cell simulations [8]. These features have been attributed mainly due to the relativistic effect. At weakly relativistic light intensities $I\lambda^2 \ll 10^{18}$ W/cm$^2$ $\mu$m$^2$ (here $I$ is the laser intensity and $\lambda$ the laser wavelength), the corresponding envelope equation of laser beams can be reduced to that in Kerr-type materials. However, when $I\lambda^2 \geqslant 10^{18}$ W/cm$^2$ $\mu$m$^2$, each beam will produce significant density depression along their propagation axes due to the transverse ponderomotive force of laser beams [11], which cannot be neglected. The effect of this density modification on the mutual interaction of spatially separated beams has not yet been explored explicitly in earlier studies. This nonlinearity is found only in plasmas and is very important when the laser power exceeds the relativistic self-focusing threshold about $17(\omega/\omega_p)^2$ GW [11], where $\omega$ and $\omega_p$ are the laser frequency and electron plasma frequency, respectively.

In this paper, we present numerical simulation studies on the interaction between two light beams launched into underdense plasma in the direction parallel to each other. We solve a set of coupled envelope equations numerically with both the relativistic nonlinearity and the ponderomotive-force effect taken into account. We demonstrate that under some circumstances, two beams can merge each other, split into three beams, or remain to be trapped in the density channel. Hosing propagation owing to the mutual interaction is observed.

In the slowly varying envelope approximation, the coupled evolution equations for two laser beams copropagating in underdense plasma can be written as [11,15]

$$2i\frac{\partial a_1}{\partial \tau} + \nabla_\perp^2 a_1 + (1 - n/\gamma)a_1 = 0, \tag{1}$$

$$2i\frac{\partial a_2}{\partial \tau} + \nabla_\perp^2 a_2 + (1 - n/\gamma)a_2 = 0, \tag{2}$$

which describe the beam propagation in a comoving frame $\xi = x - (k_0 c^2/\omega_0)t$. Here $a_1$ and $a_2$ are the slowly varying vector potentials of the two beams normalized by $mc^2/e$, respectively, the relativistic factor $\gamma = (1 + |a_1 + a_2|^2)^{1/2}$, the density $n = \mathrm{Max}(0, 1 + \boldsymbol{\nabla}_\perp^2 \gamma)$ addressing the ponderomotive expulsion of electron density from the high intensity regions, which is normalized by the unperturbed plasma density $n_0$. Also here $\tau = \omega_p^2 t/\omega_0$, with $\omega_0$ the frequency of laser beams and $\omega_p = (4\pi n_0 e^2/m)^{1/2}$ the plasma frequency, $\boldsymbol{\nabla}_\perp^2 = \partial^2/\partial y^2 + \partial^2/\partial z^2$ with transverse coordinate $y$ and $z$ normalized by $c/\omega_p$. We assume that the two beams are parallel polarized. The coupled Eqs. (1) and (2) can describe the mutual interaction of two beams through the nonlinear effects including

 

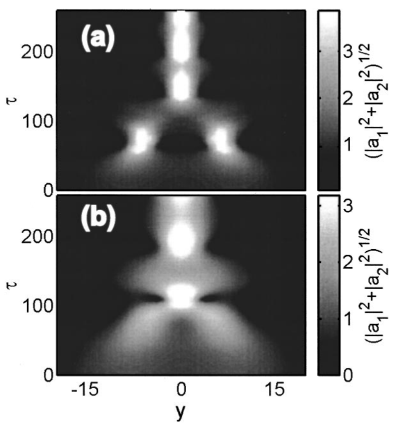


FIG. 1. Evolution of the absolute amplitude of two beams $(|a_1|^2+|a_2|^2)^{1/2}$ with initial parameters $a_{01}=a_{02}=0.6$, $\rho_{01}=\rho_{02}=8$, $y_{01}=10$, and $y_{02}=-10$. (a) Taking into account the electron-density modification due to the ponderomotive force; (b) neglecting the electron-density modification.

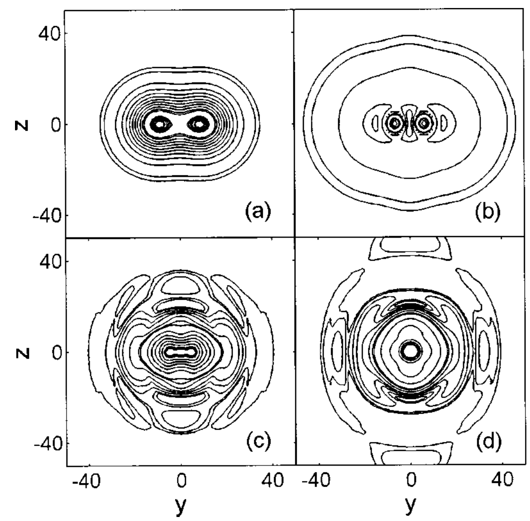


FIG. 2. Snapshots of the transverse section of the beam profile at $\tau=0$ (a), 70 (b), 110 (c), and 170 (d). The initial parameters are the same as in Fig. 1, taking into account the electron-density modification.

the relativistic nonlinearity and the density modification by the transverse ponderomotive force. However, certain kinetic effects, such as electron acceleration, attraction of electron filaments, and corresponding quasistatic magnetic generation, etc. [16,17], have been neglected. These are known to contribute to the merger of light filaments. Usually, these effects are significant in plasma with moderate densities, but relatively weak in tenuous plasma [18,19]. Thus the results described following should apply preferably in tenuous plasma such as $n_0/n_c<0.01$, where $n_c$ is the critical density of incident laser beams. In addition, since we have neglected the longitudinal profiles of laser beams, our results should apply to the case when the durations of the laser beams are much longer than a plasma oscillation period.

Equations (1) and (2) have been solved with the algorithm of the alternating-directing implicit method [15]. A rectangular simulation box is used in the *y-z* plane. In the simulations, the input beams are lunched along the x direction; the transverse beam profiles are in Gaussian with $a_1=a_{01}\exp\{-[(y-y_{01})^2+z^2]/2\rho_{01}^2\}$ and $a_2=a_{02}\exp\{-[(y-y_{02})^2+z^2]/2\rho_{02}^2\}$. With these, the normalized threshold power for relativistic self-focusing for individual beams is reached when $a_{01}^2\rho_{01}^2\geqslant 16$ and $a_{02}^2\rho_{02}^2\geqslant 16$. One notes that it is important to study the beam interaction in rectangular geometry rather than in slab geometry, so that one could compare the simulation results with real experiments; in slab geometry, there is not any power threshold for self-focusing [21].

Our simulations show that the interaction of two beams displays a variety of interesting features such as attraction, fusion, fission, and beam hosing. Some of them are similar to those found in earlier studies in nonlinear optical materials, while some of them are distinctive owing to the nonlinearity related to the transverse ponderomotive force of light beams in plasma. Figure 1(a) illustrates the evolution of the two beams when $a_{01}=a_{02}=0.6$, $\rho_{01}=\rho_{02}=8$, $y_{01}=10$, and $y_{02}=-10$. The two beams start to self-focus individually in the earlier stage. Meanwhile, they appear to attract each other. Around $\tau=70$, they are focused to the minimum spot size and then begin to defocus. After $\tau=100$, the two beams begin to merge into a single beam very quickly, which remains self-focused as a single beam afterwards. Figure 1(b) shows the evolution of the two beams for the same parameters as in Fig. 1(a), except for ignoring the density depression caused by the transverse ponderomotive force, i.e., let $n=1$ instead of $n=\mathrm{Max}(0,1+\nabla_\perp\gamma)$. In this case, the two beams merge into a single beam more quickly than that in Fig. 1(a). One notes that the final beam-spot size is smaller in Fig. 1(a) than in Fig. 1(b), demonstrating that the ponderomotive force helps to trap the light beam. One also notes that, during merging process, there exists strong emission of radiation in both cases. This emission appears to be much stronger than that when there is only one laser beam self-focusing in plasma [22]. This is more obvious in Fig. 2 showing snapshots of the transverse section of the incident beams at different times. One observes that, after merging into a single beam, its transverse section is nearly isotropic in *y-z* plane in the central region. This suggests that a rounded beam is more stable than other shaped beams in this case.

Figure 3 shows the beam evolution at a higher laser intensity and a higher beam power when $a_{01}=a_{02}=1$, $\rho_{01}=\rho_{02}=8$, $y_{01}=10$, and $y_{02}=-10$. In this case, electron-density modification is much stronger than that for Fig. 1(a). During the earlier stage, mutual attraction of beams is found while the two beams are undergoing self-focusing individually. Afterwards, the beam intensities around the beam center regions increase significantly. As a result, the electron-density depression gets deeper around the individual beam axes until electron-density cavitation occurs. These density cavities trap the two beams, preventing them from merging into a single beam. The centroids of the two beams are shifted, respectively, from their original positions due to their mutual attraction. Strong emission of radiation is found before the trapping process. However, after the beams are fully trapped in their density cavities, there is almost no new emission of radiation from the trapped beams as shown in Fig. 3(a). Figure 3(b) shows snapshots of electron-density distri-

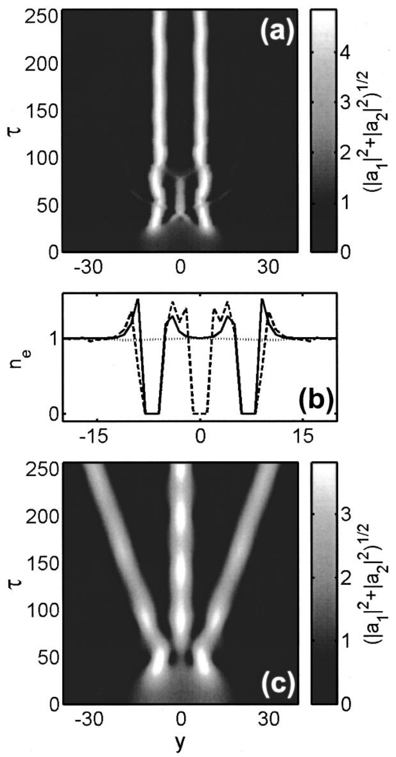


FIG. 3. Evolution of the absolute amplitude of two beams $(|a_1|^2+|a_2|^2)^{1/2}$ with initial parameters $a_{01}=a_{02}=1$, $\rho_{01}=\rho_{02}=8$, $y_{01}=10$, and $y_{02}=-10$. (a) Taking into account the electron-density modification due to the ponderomotive force; (b) electron-density profiles cut at $z=0$ at $\tau=0$ (dotted line), 50 (dashed line), and 150 (solid line); (c) beam evolution when ignoring the electron-density modification.

butions along $y$ axis cut at $z=0$. If one excludes the electron-density modification by the ponderomotive force, the beam evolution is illustrated in Fig. 3(c). It differs from Fig. 1(b) for the case at lower incident power as well as from Fig. 3(a)—the two beams neither merge into a single beam nor remain individual self-focusing. In the earlier stage, the two beams attract each other as usual. At certain time around $\tau=60$, rather than merging into a single beam, the two beams split into three beams; one beam at the center propagates along the original incident direction of the two beams, while the other two beams propagate obliquely. This appears to be a more stable state than merging into a single beam. This suggests that the density depression caused by the ponderomotive force plays role in intense multibeam interaction in plasma. Figure 4 shows snapshots of the transverse section of the incident beams when the ponderomotive force is taken into account, corresponding to Fig. 3(a). Strong emission of radiation is found around $\tau=90$ in Fig. 4(c).

If one increases the initial distance between the centroids of the two beams, it is expected that their mutual interaction becomes weaker and the two beams would remain as individual. Figure 5 shows the evolution of the two beams for $a_{01}=a_{02}=1$, $\rho_{01}=\rho_{02}=8$, $y_{01}=12$, and $y_{02}=-12$, i.e., with larger displacement than that for Fig. 3. In earlier stage, the mutual attraction is still found while the two beams are undergoing self-focusing individually. Afterwards, however,

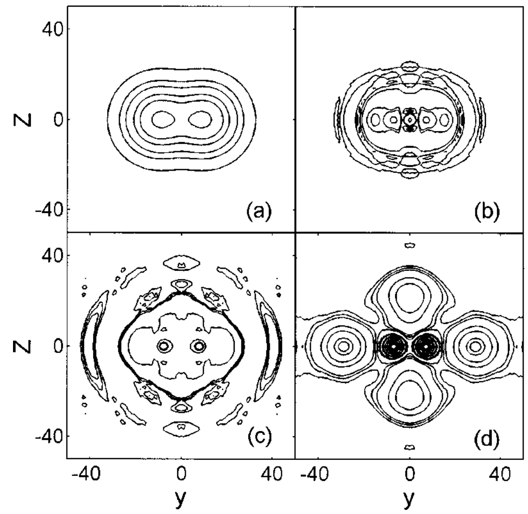


FIG. 4. Snapshots of the transverse section of the beam profile at $\tau=0$ (a), 50 (b), 90 (c), and 190 (d). The initial parameters are the same as in Fig. 3 taking into account the electron-density modification.

rather than propagating straight forward in the density channels, the two beams change their propagation directions continually, i.e., the trajectories of both beams becomes unstable. This hosinglike instability is caused by both the

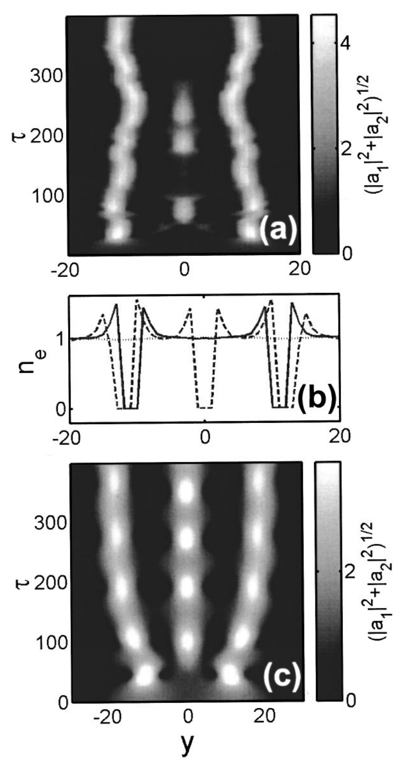


FIG. 5. Evolution of the absolute amplitude of two beams $(|a_1|^2+|a_2|^2)^{1/2}$ with initial parameters $a_{01}=a_{02}=1$, $\rho_{01}=\rho_{02}=8$, $y_{01}=12$, and $y_{02}=-12$. (a) Taking into account the electron-density modification due to the ponderomotive force; (b) electron-density profiles cut at $z=0$ at $\tau=0$ (dotted line), 70 (dashed line), and 300 (solid line); (c) beam evolution when ignoring the electron-density modification.

mutual attraction and density cavitation that prevents two beams from merging into a single one. As a result, it has a different physical origin from the normal hosing instability when a laser propagating in underdense plasma [20], which is caused by upward or downward tilting of the local wave fronts due to the transverse phase velocity difference across the wave front. This kind of instability cannot be observed for a single laser beam propagating in plasma in our simulations working in a comoving frame. As in earlier examples, we have simulated the beam envelope evolution for the same parameters, except for excluding the ponderomotive force. In this case, similar to the case for Fig. 3(c), one finds that two beams split into three beams after a self-focusing stage at the beginning. In these three beams, one propagates straight forward along the initial direction of incident beams, while the other two propagate at some angles from the initial propagation axis. Later on, they bend toward the central beam owing to mutual attraction.

In conclusion, the interaction of two copropagating light beams in underdense plasma has been studied numerically. Beam fusion/mergence, fission, and hosing during the propagation are observed. It shows that the relativistic nonlinearity can lead to beam fusion and fission, while the electron-density cavitation due to the transverse ponderomotive force of light beams tends to prevent the beam from merging into a single beam for laser beams at high intensities. The mutual interaction may also cause hosing propagation of beams. Strong emission of radiation is found during beam fusion process.

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